Gathematics (Maths in the Gas Industry)

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The gas industry uses basic mathematics, much as any other industry, for accounting functions, wage allocation, budgeting and other finance related areas. However here we will discuss a few applications of mathematics specific to the gas industry. These are calculations of

(a) pressure drops in pipes and

(b) standard volumetric flowrates.

(a) Newcastle receives its natural gas, via Sydney, from Moomba in South Australia (see Figure 1) where the pressure is about 7000 kPa[1]. When the gas reaches the outskirts of Sydney it can take one of several routes. Some continues on to feed the city centre, some goes south to Wollongong and even Canberra - the rest travels north to feed Newcastle (see Figure 2). By the time the gas reaches Hexham Trunk Receiving Station (T.R.S.) the pressure has dropped to below 5000 kPa. These pressure drops are caused by friction between the gas and the walls of the pipe and by the fact that gas cannot be replaced as quickly as it is used. Many equations have been proposed that predict these pressure drops but the one favoured by the gas industry is

$$P_1^2 - P_2^2 = 3.6 \times 10^7 \cdot \frac{Q^2 L}{F^2 D^5}.$$

where

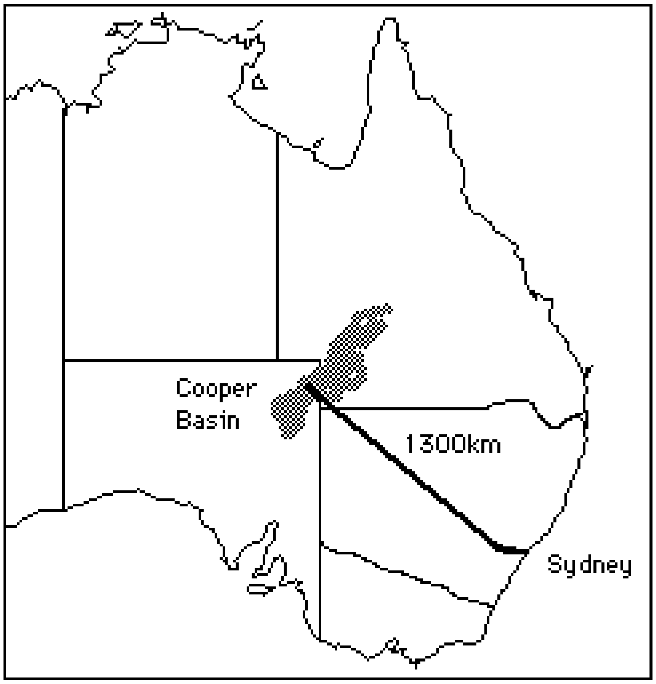
P_1 and P_2 are the inlet and outlet pressures in kPa,

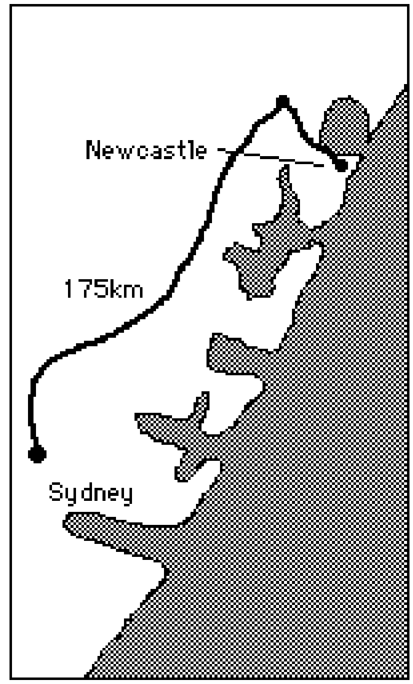
Q represents the flowrate of gas in m^3/hr,

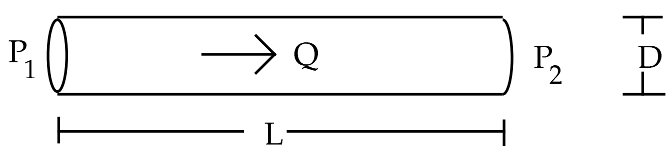
L is the length of the pipe in metres,

D is the pipe diameter in mm and finally

F is the friction factor.







The friction factor is obtained from the split definition

$$F = \begin{cases} \frac{\sqrt{Re}}{8} & \text{if } Re \leq 2000 \\ -2\log\left(\frac{0.058}{D} + \frac{7.85F}{Re}\right) & \text{if } Re > 2000 \end{cases}$$

where $Re = 32000Q/D$ is called the Reynolds Number. Note that the second equation has F on both sides and so must be solved by some approximation method e.g. Newtons method, linear iteration or the bisection method. At present the maximum flowrate to Sydney is about $285000m^3/hr$ and the diameter of the pipe from Moomba to Sydney is 860mm. Thus $Re = 1.06 \times 10^7$ which is clearly greater than 2000 so to find F we must use the second of the defining equations. Using linear iteration we find that $F = 18.8$. So now if we assume a

[1] 1 kPa = 1000 pascals = 1000 newtons/sq. m = 1000 kg/m/s/s

pressure of 7000 kPa at Moomba then the pressure at Sydney (1300km away) is given by

$$P_2 = \sqrt{P_1^2 - \frac{3.6 \times 10^7 Q^2 L}{F^2 D^5}} = 5120kPa.$$

We can now use this resultant pressure as the inlet pressure on the line from Plumpton T.R.S to Hexham T.R.S. The maximum flowrate to Hexham T.R.S. is about $60000m^3/hr$ but the pipe diameter has been reduced to only 500mm. So now $Re = 3.84 \times 10^6$ which leads to a friction factor of $F = 17.6$. Now assuming the pressure at Plumpton T.R.S. is $P_1 = 5120kPa$ then the pressure at Hexham T.R.S. is given by

$$P_2 = \sqrt{P_1^2 - \frac{3.6 \times 10^7 Q^2 L}{F^2 D^5}} = 4886kPa.$$

About half of the gas that arrives at Hexham T.R.S. is used there while the other $30000m^3/hr$ continues on to Newcastle. It is left as an exercise to the reader to calculate the final pressure drop given that the diameter of pipe is now only 355mm and the length is about 12 km. Because of industrial process limitations the minimum tolerable pressure at Hexham T.R.S. is 3700 kPa. So if we again assume a pressure of 5120 kPa at Plumpton T.R.S. and use the same friction factor (justified later) we find that the maximum allowable flowrate at Hexham T.R.S. is given by

$$Q_{max} = \sqrt{\frac{(P_1^2 - P_2^2)F^2 D^5}{3.6 \times 10^7 L}} = 138722m^3/hr.$$

To justify the use of $F = 17.6$ note that if we now use Q_{max} to calculate a new Reynolds number and hence a new friction factor, F' say, we find that $F' = 17.87$. Using F' in the above equation we can calculate a new maximum flowrate $Q'_{max} = 140850m^3/hr$ with which we can continue the iterative reasoning until both Q_{max} and F converge to some fixed values. These turn out to be $F = 17.872$ and $Q_{max} = 140866m^3/hr$. Thus Newcastle is using only about 43% of the full capacity of the trunk main. The versatile pressure drop equation is also used to predict the necessary diameter of pipe required to supply say a new industry or a new domestic area. However we must know beforehand the inlet pressure, an estimate of Q_{max}, the minimum tolerable pressure at the end of the pipe and the distance from the supply point. Since D is required in the formula for F we have to resort to an iterative argument similar to the one used above.

(b) As we saw in the first section the pressure of gas varies from point to point in a pipe network. Now recalling the ideal gas law

$$PV = nRT$$

where P is the pressure, V is the volume, T is the temperature[2] and n and R are constant. If we fix the temperature then as pressure varies the volume will vary. Similarly if pressure is constant then as temperature varies the volume will again vary. This makes the measurement of the volume of gas, bought or sold by the gas industry, a rather tricky procedure. To get around the problem, of varying pressures and temperatures, whenever volume is measured the pressure and temperature of the gas are also measured and the volume is the "adjusted" to base conditions. This is a temperature of 288 K and a pressure of 101.3 kPa. If we let the subscript "b" stand for base conditions and the subscript "f" for flowing conditions then

$$P_b V_b = nRT_b$$

and

$$P_f V_f = nRT_f.$$

Dividing these two equations and rearranging to obtain V_b at the head of the resulting equation we find that

$$V_b = V_f \cdot \frac{P_f}{P_b} \cdot \frac{T_b}{T_f}.$$

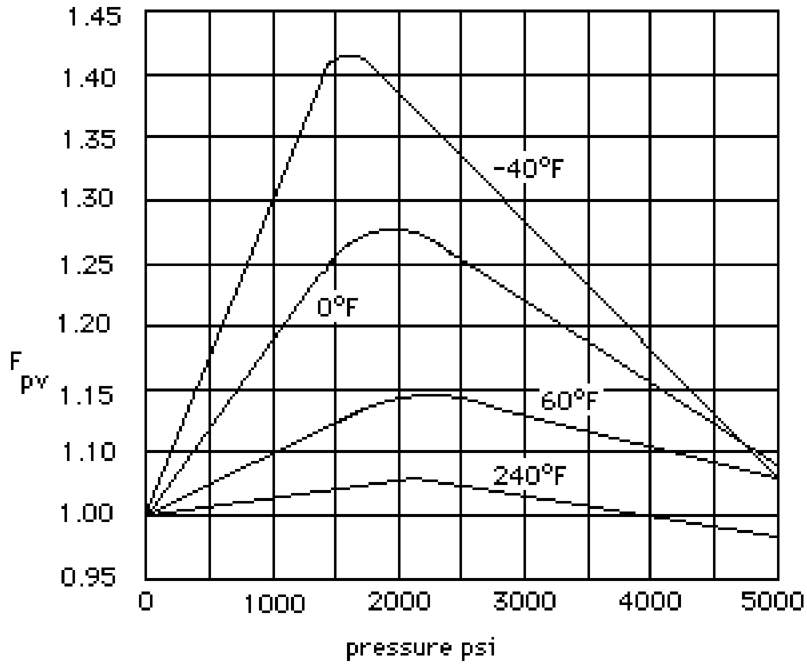
However this is only valid for an "ideal gas" and since real gases are not ideal (far from it at high pressures) the equation used by the gas industry is

$$V_b = V_f \cdot \frac{P_f}{P_b} \cdot \frac{T_b}{T_f} \cdot F_{pv}^2.$$

where F_{pv} (the so-called supercompressibility factor) is a measure of the difference between the real gas in question and an ideal gas under the same conditions. It turns out that F_{pv} depends on many factors specific to any particular gas but once the gas has been chosen (e.g. Natural Gas) the two main factors are pressure and temperature of the gas (see graph). Formulae do exist that predict the values of F_{pv} for varying pressures and temperatures, however they are beyond the scope of this paper. Note that the graph uses imperial units, where 1 psi = 6.895 kPa.

The gas company has microcomputers at several large industries which receive signals that tell them the pressure of the gas P_f, the temperature of the gas T_f, and the actual flowrate V_f. These so-called flow computers use P_f and T_f to first calculate F_{pv} and then use all four values P_f, T_f, V_f and F_{pv} to calculate the corrected flowrate V_b via the equation above. If for any reason the

[2]Note: T is in degrees Kelvin which is Celcius +273.15



flow computer stopped its calculations then a manual determination of the volume missed would have to be made. For example, suppose the flow computers at Hexham T.R.S. had failed for 24 hours during which the actual volume used was $V_f = 14000m^3$. Assuming that $P_f = 5000kPa$ and $T_f = 288K$ then from the graph we see that $F_{pv} = 1.06$ (at 725 psi, 60 F). Given that $P_b = 101.3kPa$ and $T_b = 288K$ we can calculate the corrected volume for the day to be

$$V_b = 14000 \times \frac{5000}{101.3} \times \frac{288}{288} \times 1.06^2 = 776426m^3.$$

This gives an average rate of $32351m^3/hr$ which approximates the usage mentioned in part (a) for Hexham T.R.S.