The Infinity Heresy

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1 Introduction

The fundamental point that I am trying to make in this paper is really quite simple.

Postulate 1

Infinity does not exist.

Of course, before an unending number of objections start pouring in to my bottomless email box, I need to clarify this brazen claim. After all, humans have been grappling with the concept of infinity for many millennia and mathematicians seem to have (at least in the last 150 years or so) finally tamed it.

First, building on the work of Euler, Cauchy and Weierstrass, we now have the ability to work with infinite series and sequences. We sum them, take their products and manipulate them in many ways—all with a mathematical rigour that would have made the ancient Greeks envious.

Furthermore, after Cantor's spectacular discoveries, we even give the various distinct sizes of infinities different names such as

$$\aleph_0, \aleph_1, \ldots$$

and so on—to infinity.

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From the use of continuity in calculus to induction in logic, infinity is implicitly present and this paper will not seek to overturn such spectacularly successful applications.

However, while the concept of infinity as a process is one thing it is completely another to blithely assume that actual infinities exist and can be manipulated in the real world. It is here in the real world where the postulate applies—and lest anyone think that this is irrelevant sophistry there are falsifiable consequences. To make it possible to discuss such consequences it is useful to modify the vague boxed statement above and turn it into something we can all argue about.

Postulate 2

An actual instantiation of infinity does not exist in the observable universe.

Two consequences are immediate.

- No computer will ever be able to add two random real[1] numbers.
- Continuity will always be an approximation to reality.

2 Computers can't add

While computers seem to be able to do almost anything, that their programmers program them to, they are fundamentally unable to do certain things. My claim (and you should take this as a challenge) is that no computer will ever be able to add two random real numbers together. This remains true whether it is a Turing machine, a quantum computer, or one based on string theory or M-branes or whatever happens to be your current favourite ultimate theory of the universe.

The way a computer adds numbers together is that it takes a finite representation and in a finite number of operations (basically glorified kindergarten addition) produces the digits of the sum. Here is an example I prepared earlier

$$1 + 1 = 2.$$

[1] To remove any ambiguity, this use of the word *real* refers to numbers from the set $\mathbb{R}$.

Computers can do pretty well with integers, rational numbers and algebraic numbers but that is where their accuracy ends. The real number system ($\mathbb{R}$) also contains delightful beasts called transcendental numbers [9]. These have the property that when they are expressed in decimal form they are infinite and non-repeating. Note that this does not mean that any non-repeating decimal is transcendental. For example, the square root of two is a non-repeating decimal

$$\sqrt{2} = 1.41421\ 35623\ 73095\ 04880\ 16887\ \ldots$$

but it is not transcendental. In fact it has a finite description as a root of a polynomial with integer coefficients. It is this finite representation that is manipulated by computers to produce exact results for any algebraic number.

Transcendental numbers, by definition, do not satisfy a polynomial with integer coefficients. But what is even more interesting is that (apart from a very small set) they have no possible finite description.

There is a subtlety here that some may think will lead to an escape route. While $\sqrt{2}$ is non-repeating when expressed as a decimal, if it is expressed as a continued fraction it becomes repeating

$$\sqrt{2} = 1 + \frac{1}{2 + \frac{1}{2+\frac{1}{2+\ldots}}}$$

almost immediately. One can clearly see how the pattern of twos ought to continue, a fact which when proven allows us to provide a very succinct description of the square root of two. So we may hope that we can do something analogous for transcendentals by somehow choosing a different or cleverer representation which will save the day *i.e.* provide them with finite descriptions. However, this hope is doomed to failure.

The problem with transcendentals is their incompressibility—there is simply no shorter description of such a number than that obtained by writing it out in full as a decimal. Since most real numbers are transcendental, when you choose two random real numbers you will almost always choose two transcendentals.

To sum two transcendental numbers, like π, e, or Liouville's constant[2], the best we can do is either pretend, by using symbolic manipulation such as

$$\pi + \pi = 2\pi,$$

[2] The first decimal number ever proven to be transcendental.

or make an approximation like

$$\pi + \pi = 6.28318\ 53071\ 79586\ 47692\ 52867\ \ldots$$

and tactfully placing dots to highlight our obvious inability to complete the task. The former attempt doesn't really constitute a calculation since it merely juxtaposes symbols and hence this approach cannot answer most questions one would ask without resorting to an approximation. The latter attempt makes it painfully clear why we are failing. Since no computer has infinite storage space it is impossible to even read in the numbers let alone add them.

3 The Universe—Continuous or Discrete

Since the time of Zeno of Elea and his paradoxes there has been almost continuous not-so-discreet debate on the nature of the universe in which we live. Is it continuous or discrete, or both, ... or neither?

When Newton and Leibniz invented calculus it implicitly carried within it the seeds of continuity. As a result of the spectacular utility of this new method to effortlessly solve problems that were essentially intractable to an earlier age it became very easy to believe that there was a direct correspondence between the universe and continuity. Of course, if the granularity of the universe is so fine that only very subtle and delicate experiments can discern it then it is easy to be lulled into a false sense of security about such a correspondence.

I would argue that in many cases a continuous calculation, like an integral, is far easier to evaluate than an analogous discrete one, like a finite sum, simply because the sum involves too many terms to complete. After all, who among us can actually calculate

$$1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{\text{googol}}$$

despite the fact that we know it to be nothing other than a rational number[3]. So my claim is that the last 300 years of calculus have been an interesting

[3] The astounding fact that the closely related harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, was proven by Nicole d'Oresme sometime between 1323 and 1382 [7].

diversion which will ultimately be seen as a useful approximation but not really corresponding to the universe in a fundamental way.

Interestingly, while mathematicians have been refining their tools to tame the infinite, physicists have been amassing evidence in favour of the notion that the universe is really discrete.

No physical experiment will ever measure a quantity that is infinite in magnitude. Furthermore no quantity that takes a transcendental value can be measured with complete precision. Either of these tasks would take infinite accuracy, infinite time or infinite storage to achieve—and postulate 2 precludes this.

Of course, just because we can't make these measurements doesn't mean that the universe necessarily fails to be continuous. However, there is much evidence to support the concept that the universe is discrete. In its most extreme form this means that all quantities:

space, time, mass, charge, energy, spin, temperature, pressure, ...

are discrete. Evidence for this includes an anomaly, a bound, a catastrophe, a dilemma and an experiment.

3.1 Ultrahigh energy cosmic ray anomaly

In 1966 three Russian theorists [6] [17], used Einstein's theory of special relativity (which is based on an underlying assumption that space is continuous) to predict that cosmic rays impinging on the upper atmosphere of the earth would have no more than a certain maximum energy (5×10^{19} eV). This calculated maximum energy, now called the GZK bound, is apparently due to the fact that a particular interaction with the background microwave radiation (a remnant of the Big Bang) absorbs any cosmic rays with excess energy. Subsequent experiments confirmed the impossible—namely, that cosmic rays were actually observed with more energy than the GZK bound allowed. Around the turn of the millennium it was proposed [12] [11], that if space were discrete this would allow higher energy cosmic rays to reach the earth and hence resolve the anomaly.

3.2 Bekenstein bound

A remarkable result [2] [1], due to the application of the Heisenberg uncertainty principle to black holes, reveals that there is a bound on the number of quantum states (and hence information) that can exist inside a spherical region of space. The bound is given by

$$I_{max} = CMR$$

where M is the mass inside a region of radius R and $C = 4\pi^2 c/(h \log 2)$ or about 2.57686×10^{43} bits/(m-kg). If we apply this to a proton we find that it can contain no more than 32 bits worth of information. We can do much better than this by packing more matter into the same space. However, if we restrict ourselves to using no more space than that contained in a proton, then the best we can possibly do is to compress enough matter[4] into it until it becomes a black hole. The Bekenstein bound would then limit this minuscule behemoth to at most about 2^{134} bits.[5] While this does imply that current computers are vastly underutilising this enormous potential storage, what is surprising about the Bekenstein bound is that it is finite. Even allowing for the well documented parallelism gained from superposition of quantum states one can never store more than

$$2^{134} = 2\ 17780\ 71482\ 94006\ 16616\ 55974\ 87563\ 31655\ 33184$$

distinct values in a region the size of a proton. Of course, such a finiteness result applies to any finite spherical region of the observable universe.

3.3 Ultraviolet catastrophe

All matter in the universe absorbs and emits energy in the form of electromagnetic radiation. A *blackbody* is an ideal object with the property that it absorbs all incident radiation and radiates the maximum possible amount of energy at a given temperature. When a blackbody is in thermal equilibrium with its environment then classical physics (with help from Rayleigh

[4] Setting the Schwarzchild radius ($R = 2GM/c^2$) equal to the proton radius one finds that 8.08×10^{11} kg or about 130 Great Pyramids is just enough matter. One can imagine constructing the requisite number of pyramids, but their compression would be a little tricky.

[5] For comparison, current 160 GB hard disks contain about 2^{40} bits worth of storage.

and Jeans) predicted that the energy radiated increases without bound as the frequency increases. Since this patently does not occur for real objects, like stars or cavities, this departure of reality from theory is known as the ultraviolet catastrophe.

As it turned out the fundamentally flawed assumption was that energy was a continuously variable quantity. When Planck postulated that energy came in discrete packets, called quanta, it led to a spectacular resolution of the dilemma and helped usher in quantum theory—a theory in which the discrete rules.

3.4 Banach-Tarski paradox

Three dimensional Euclidean space ($\mathbb{R}^3$) has some counterintuitive properties. For example, Banach and Tarski showed [15] that one can dissect a solid sphere in $\mathbb{R}^3$ into 5 pieces and reassemble them, using only rotations and translations, into a new solid sphere of twice the volume. It is curious that, in almost any other field of mathematics, when one is faced with a situation like

$$1 = 2$$

the usual response is to claim a contradiction has occurred and subsequently point to the incorrect assumption. While terms like "dissect", "piece", and "reassemble" have precise mathematical descriptions they are derived from real life analogs. Since everyday experience clearly demonstrates that such an event does not occur[6]—it should be clear why this is called a paradox.

Now which of the assumptions underlying the proof of the Banach-Tarski theorem causes this model to deviate so wildly from reality?

The usual explanation involves their use of the Axiom of Choice which states that one can always choose precisely one element from a collection of sets. This sounds innocuous, especially when applied to finite collections of finite sets—however, all hell breaks loose (at least amongst mathematicians) in the attempt to apply it to infinite sets.

An equally valid resolution of the problem is that $\mathbb{R}^3$ is continuous while this is not necessarily true for the universe. In fact, if the universe is discrete

[6]Unless it happens to be the driving force behind the expansion of the universe.

then the Banach-Tarski Theorem cannot be applied. Either way ∞[7] is the culprit.

3.5 Stern-Gerlach experiment

A rotating object (like a planet, or gyroscope) has a property called angular momentum which is a vector quantity pointing along the axis of rotation. Atomic particles have an analogous property called spin angular momentum. While a macroscopic object can presumably spin in such a way that its axis points in any direction in space, the same is not necessarily true of atomic and subatomic particles.

In 1922 Stern and Gerlach fired silver atoms through an inhomogeneous magnetic field in an attempt to falsify either classical physics or quantum physics since each predicted a significantly different result. On the one hand classical physics predicted a bright spot of silver atoms in the centre of the collector plate while quantum physics predicted two bands either side of the central beam path. In the event they demonstrated that the direction of the spin of angular momentum was quantized in space into only two directions—up or down. Since the entire universe is permeated by magnetic fields this has the somewhat disturbing implication that I cannot point my finger in arbitrary directions in space.

This was in fact the first experiment which suggested the discretization of space.

Exercise 1 *Assuming postulate 2 prove that:*

(a) circles do not exist,

(b) there is no singularity at the centre of a black hole,

(c) God does not exist.

[7] The first use of this well-known symbol for infinity was in 1665 by John Wallis in his Arithmetica infinitorum [16].

4 Manifestations of the concept of infinity

Various human endeavours, either implicitly or explicitly, contain examples of the concept of infinity. The common thread of these examples is that they invariably define some sort of boundary between what is possible and what will always remain just beyond our reach.

4.1 Astronomy—Black Hole

When a sufficiently large stellar mass reaches the point in its life-cycle that the inward pressure of gravitational attraction is greater than the outward photon pressure, the ensuing supernova is predicted (under some very plausible and realistic assumptions [10] [8] [3]) to generate a black hole. This is simply an object for which the gravitational strength is so great that light originating inside the event horizon cannot escape to the rest of the universe. So far there is nothing really surprising going on—unless you are a 10 year old child hearing this for the first time. However, there is at present no known force that can withstand the continuation of the stellar collapse to the point of a singularity. All the mass of the black hole is supposed to reside at a single point in space—creating, amongst other things, an infinite density. This is often seen as a jumping off point for most space travellers on their way to Andromeda, however most physicists realise that it reveals a breakdown in the two theories, general relativity (GR) and quantum mechanics (QM), used to describe the situation at hand. While there is currently plenty of astronomical evidence for black holes–there is little supporting a singularity in any one of them. A possible resolution of this singularity may come from a new theory superceding QM and GR.

4.2 Computer science—Oracle

In the spirit of the ancient Delphic Oracle, the modern analog is simply defined to be an imaginary device that provides the (necessarily correct) answer to any decision problem in a single step. These are typically used in the theoretical analysis of the complexity of algorithms and I used to be of the opinion that resorting to such an artifice was, well, artificial. The fundamental point that I continually missed was that we can approximate an oracle by simply creating a parallel computer with enough processors so that each one can work on each possible input to the problem. This works very well if there are a finite number of inputs to our decision problem—however it breaks down when there are infinitely many inputs. Thus an oracle defines a boundary for what is computationally feasible, even if all the quantum states of the observable universe were at our disposal.

4.3 Cosmology—Big bang

The work of Hubble and his successors showed that the galaxies[8] are currently moving away[9] from each other. By inference, they must have been closer together at some time in the past. Not only that, but the evidence of the spectrum of the uniform background radiation (at microwave frequencies) together with the apparent light element abundances suggest that the universe began as an extremely compact object. The implied origin of the universe in the Big Bang implies, just as the black hole does, that there was a singularity at the creation. What this really points to is the same incompatibility between GR and QM mentioned earlier. This time the resolution of the difficulty may come from Hawking's no-boundary boundary condition which allows time to become space-like and actually smear out the singularity.

4.4 Mathematics—Circles

There are far too many instances of infinity in mathematics to discuss in this short paper. However, to take a simple one we consider a familiar object, the everyday, garden variety, circle. To answer the question "Does a circle exist?" will depend on your definition of "does", "a", "circle" and "exist". I will leave the words "does", "a" and "exist" for the philosophers and linguists to argue over (after all, they need something to do). If, as is usually the case, we define a circle to be the collection of points in $\mathbb{R}^2$ which are at a fixed

[8]When they were first discovered, galaxies were called "island universes" since the Milky Way was the entire universe and these new collections of stars were clearly outside the Milky Way. It took a while for us to recall that the word universe is synonymous with everything and as a result has to expand to fit the current viewpoint. A similar effect occurred with multiple universes at the turn of the second millennium.

[9]The current viewpoint is that it is actually space itself that is expanding and carrying the galaxies along with it.

distance from a given point then this set is infinite in size. Not only that but each point itself is an infinite object since it is comprised of a pair of real numbers which themselves are usually transcendental. Neither such points, nor an infinite collection of such points exist in the universe. Furthermore, such a set of points does not exist in the wetware of any sentient being, since these are finite constructs. We could take the pragmatic view and define the circle to be synonymous with its Cartesian equation which we then can write down using only a finite number of symbols—for which we could make a claim of existence.

4.5 Mathematics—Induction

Induction is a remarkable process [4] which allows one to prove an infinite number of propositions using only a finite number of statements. The archetypal example stems from the following pattern:

$$\begin{aligned} 1 &= 1 \times 2/2 \\ 1+2 &= 2 \times 3/2 \\ 1+2+3 &= 3 \times 4/2 \\ 1+2+3+4 &= 4 \times 5/2 \\ 1+2+3+4+5 &= 5 \times 6/2 \\ &\vdots \end{aligned}$$

to which anyone would sensibly respond that the sum of the first 100 integers is just $100 \times 101/2$ rather than add them up. However this observation of a pattern does not constitute a proof since finite patterns can fail to continue. Of course one can certainly make a guess that the sum of the first n integers is given by

$$S(n) : 1+2+3+4+...+n = \frac{n(n+1)}{2},$$

however there is no notion of a proof in this transfer of information from pattern to general formula. The way a proof does emerge is to show the truth of precisely two things, namely,

the initial condition : $S(1)$, and

the inductive step : $S(n) \to S(n+1)$.

Once the inductive step is proven using a finite number of statements then we can simply bootstrap the process and infer that

$$S(1) \to S(2) \to S(3) \to S(4) \ldots$$

and hence that the pattern does in fact continue forever. Unlike the deductive process, induction has no analog in the observable universe. There is simply no known process with the property that: if you know it occurs for a general value of an associated parameter this allows you to prove that it must also occur when the parameter is increased by one.

It turns out that the Principle of Induction is equivalent to the so-called Well Ordering Principle which in turn is equivalent to the Axiom of Choice. Since we felt squeamish about this axiom earlier, during its application to the Banach-Tarski paradox, we must take great care when attempting to apply the results of inductive proofs to the universe.

4.6 Philosophy—Existence

In the 19th century Richard Dedekind wrote a delightful treatise [4], titled[10] "Was sind und was sollen die Zahlen?", in which he gives a proof of the existence of infinite systems. He claims that the collection, S say, of all possible things that can be the object of his own thought processes is infinite. He considers the properties of an ingenious mapping, $\phi : S \mapsto S$, operating on this set and given by

$$\phi(s) = \text{the thought corresponding to "thinking about } s\text{".}$$

for any $s \in S$. Clearly, $\phi(S)$ is a collection of thoughts and is thus contained in S. Furthermore, Dedekind makes the observation that $\phi(S)$ is in fact a *proper* subset of S, since S contains thoughts not contained in $\phi(S)$—amusingly offering his own ego as an example. Finally, he notes that $\phi(a) \neq \phi(b)$ for any two distinct elements $a, b \in S$. Thus the mapping ϕ provides a correspondence between S and a proper subset of itself—which is the hallmark of an infinite set.

Of course, the notion that a thought like

$$\phi^{googol}(eggs) = \text{thinking about(...(thinking about eggs)...).}$$

[10] "The nature and meaning of numbers."

could actually be contained in Dedekind's (or anyone's) collection of thoughts is ludicrous, not least since a tower of increasing size thoughts defined this way requires more and more energy to consider. This ultimately reaches a point where the person involved needs to consume more mass that exists in the observable universe simply to have the energy to consider the next thought in the list.

In principle, this proves the existence of an infinite set—but in principle we may as well just assume an infinite set exists. One seems to have gained little by this exercise.

4.7 Physics—Speed of light

The bound placed on speeds, by Einstein, that no object starting from rest can travel at the speed of light turns out to be associated with the infinite in a fundamental way. When one analyses the amount of energy required to accelerate particles close to the speed of light one finds that it increases without limit actually becoming infinite at c. Physicists rightly conclude that it is impossible to reach c for this reason.

Notice that this example is a rather curious one. The speed of light is a finite quantity but more importantly we can observe actual objects in the universe, namely photons, which travel at this speed.

Can photons travel faster than c?

This question is closely related to quantum mechanical tunnelling. It is possible for a photon to end up on the other side of what would normally be considered an impenetrable barrier by a process called tunnelling. The time it takes to actually pass through the barrier is less [13] than the time it would take to travel the same distance if it were travelling in free space. The naïve implication is that the photon's speed was about $1.7 \times c$. Presumably the physicists would simply explain this away by using the Bohr interpretation of quantum phenomena and claim that all we can say is that at one point in time there was a photon on the left and at a later point in time a photon on the right and it is impossible to say what happened in between.

4.8 Religion—God

> ...it is now well established that all known gods came into existence a good three millionths of a second after the universe began, rather than, as they usually claimed, the previous week ...
>
> Douglas Adams, The Hitchhiker's Guide to the Galaxy

In most religions[11] a god is an all knowing, all seeing, all powerful being. Clearly, these omnipotent properties implicitly contain within them examples of infinity. The supposed mediæval debates about the number of angels that could dance on the point of a needle were not really exercises in futility, rather they are more accurately described as discussions on whether or not angels could exist without taking up any physical space.

Tipler [14] has made a bold attempt to model god-like behaviour by allowing future intelligent life to have access to ever larger fractions of the available mass and energy of the observable universe. Of course this is at best an approximation due to postulate 2.

5 Difficulties

There are a number of difficulties with any attempt to take postulate 1 at face value. Are we really expected to believe that the universe is finite in every way? While it is true that the observable universe is finite, and always will be, the universe need not be. However there is no possible causal influence on our part of the universe from anything outside the observable universe—since no signal can travel faster than the speed of light. If there is no end to time then a finite discrete observable universe must cycle—or must it?

5.1 Recurring histories

Many philosophers, scientists and theologians have considered the possibility of a recurring, cycling or endlessly repeating universe. The argument is a deceptively simple one. If the universe is finite in size and is discrete, then given enough time the same configuration of fundamental particles, as

[11] Apart from the degenerate Greek Pantheon.

exists right now say, must occur again at some later time. At that point, all subsequent evolution of the universe will be identical. In mathematics, this has even been enshrined as the so-called Poincaré recurrence theorem[12].

Of course, if the universe is finite in spacial extent and continuous, then this would seem to offer an escape route, since particles can presumably be positioned at an infinite number of locations in space-time. So the same configuration need never occur again. In this thesis, since postulate 2 precludes this possibility, the obvious conclusion we are drawn to is that time is finite.

5.2 Beta decay

When certain atomic nuclei emit an electron and then transform into the next element up, in the periodic table ordering, they are said to have undergone *beta* decay. The modern view is that one of the "down" quarks in a neutron, transforms into an "up" quark, converting the neutron into a proton, with a neutrino particle carrying away some of the energy—in agreement with the law of conservation of energy.

It was shown by Meitner and Hahn in 1911 and subsequently verified by Ellis, Chadwick that the possible energies of the emitted electrons formed a continuous set of values.

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5.3 Bounded operators

Continuous spectrum of eigenvalues of bounded operators ... (need reference)...

5.4 Relativistic quantum mechanics

In the 1951 Dyson lectures [5] the statement is made that

> A relativistic quantum theory of a finite number of particles is impossible.

[12]Which often leads physicists into paradoxical arguments—especially when coupled with the second law of thermodynamics.

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