Mayan Calendar

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The Mayan civilization had an intricate way of naming each day of their "week" which was based on the interactions of two distinct "years" :-

(a) the Tonalamatl or Sacred Year and

(b) the Haab or Political Year.

(a) The Tonalamatl was deemed to contain 260 days, each of which was designated as one of the twenty names from Table 1. This sequence of names was repeated cyclically so that Imix followed Ahau and so on just like our seven day week. Prefixing each of these names was a numerical coefficient from 1 to 13 which also repeated cyclically - clearly out of phase with the day names. So if the first day of the Tonalamatl is 1 Imix then the thirteenth day is 13 Ben while the twentieth and twenty first days would be 7 Ahau and 8 Imix respectively. Note that since $13 \times 20 = 260$ the first day of any Tonalamatl will always have the same name and coefficient - in the above example it would always be 1 Imix. Any subsequent day will similarly remain in the same position in each subsequent Tonalamatl.

The Tonalamatl can be modelled by using modular arithmetic. Let C_1 denote the coefficient of the days in the Tonalamatl and so range from 1 to 13. Let K range from 0 to 19 where each number corresponds to a day name of the Tonalamatl i.e. $0 \equiv$ Imix, $1 \equiv$ Ik, $2 \equiv$ Akbal ... etc. Now if $C_1' : K'$ is a day

A.	Imix	K.	Chuen
B.	Ik	L.	Eb
C.	Akbal	M.	Ben
D.	Kan	N.	Ix
E.	Chicchan	O.	Men
F.	Cimi	P.	Cib
G.	Manik	Q.	Caban
H.	Lamat	R.	Ezrab
I.	Muluc	S.	Cauac
J.	Oc	T.	Ahau

Table 1: Days of the Tonalamatl

A.	Pop	K.	Zac
B.	Uo	L.	Ceh
C.	Zip	M.	Mac
D.	Zotz	N.	Kankin
E.	Tzec	O.	Muan
F.	Xul	P.	Pax
G.	Yaxkin	Q.	Kayab
H.	Mol	R.	Cumku
I.	Chen	S.	Uayeb
J.	Yax		

Table 2: Divisions of the Haab

which occurs N days after $C_1 : K$ then they are related by the formula :

$$\begin{aligned} C_1' &= (C_1 - 1 + N)(\text{mod } 13) + 1 \\ K' &= (K + N)(\text{mod } 20) \end{aligned} \tag{1}$$

So for example to evaluate what day is 422 days after 1 Imix we note $C_1 = 1$ and $K = 0$ so that

$$\begin{aligned} C_1' &= (1 - 1 + 422)(\text{mod } 13) + 1 = 6 \\ K' &= (0 + 422)(\text{mod } 20) = 2. \end{aligned}$$

Hence 1 Imix + 422 days $\equiv$ 6 Akbal.

(b) The Haab contained 365 days which are grouped into 19 divisions (roughly equivalent to our months) which are shown in Table 2.

The first 18 divisions were called uinals and contained 20 days each. The last division called Uayeb contained the last 5 days of the Haab. Each day of any uinal was prefixed by a number from 0 to 19 depending on its position in the month - while the days of the Uayeb were just prefixed by numbers from 0 to 4. So as with the sacred year any day of one particular Haab would always have the same name and numerical prefix in subsequent political years. As before the Haab can be modelled by use of modular arithmetic.

Let C_2 denote the coefficients of the divisions of the Haab and so range from 0 to 19.

Let U range from 0 to 18 where each number corresponds to a division of the Haab i.e. : $0 \equiv$ Pop, $1 \equiv$ Uo, $2 \equiv$ Zip, ... , $18 \equiv$ Uayeb.

Now if $C_2' : U'$ is a day which occurs N days after $C_2 : U$ then these are related by :

$$\begin{aligned} C_2' &= \{(20U + C2 + N)(\text{mod } 365)\}(\text{mod } 20) \\ U' &= \left[\frac{(20U + C_2 + N)(\text{mod } 365)}{20} \right] \end{aligned} \tag{2}$$

where $[x]$ is the greatest integer less than or equal to x.

1.	4 Ahau 8 Cumku	11.	1 Oc 18 Cumku
2.	5 Imix 9 Cumku	12.	2 Chuen 19 Cumku
3.	6 Ik 10 Cumku	13.	3 Eb 0 Uayeb
4.	7 Akbal 11 Cumku	14.	4 Ben 1 Uayeb
5.	8 Kan 12 Cumku	15.	5 Ix 2 Uayeb
6.	9 Chicchan 13 Cumku	16.	6 Men 3 Uayeb
7.	10 Cimi 14 Cumku	17.	7 Cib 4 Uayeb
8.	11 Manik 15 Cumku	18.	8 Caban 0 Pop
9.	12 Lamat 16 Cumku	19.	9 Ezrab 1 Pop
10.	13 Muluc 17 Cumku	20.	10 Cauac 2 Pop

Table 3: First 20 days of the Maya Calendar

Then for example a day which is 422 days after 0 Pop (for which $C_2 = 0$, $U = 0$) can be found by :

$$\begin{aligned} C_2' &= \{(0+0+422)(\text{mod } 365)\}(\text{mod } 20) \\ &= 57(\text{mod } 20) \\ &= 17. \end{aligned}$$

While

$$\begin{aligned} U' &= \left[\frac{(0+0+422)(\text{mod } 365)}{20}\right] \\ &= \left[\frac{57}{20}\right] \\ &= 2. \end{aligned}$$

Hence 0 Pop + 422 days $\equiv$ 17 Zip.

The Maya now chose to name any particular day by first specifying its number and name from the Tonalamatl and then appending to this the number and name from the Haab. They chose as their starting date 4 Ahau 8 Cumku so that the next 19 days ran as in Table 3.

Note that the date 8 Caban 0 Pop is the beginning of a new Haab, since it is the first day after the 5 days in Uayeb of the previous Haab. This meshing together of the Tonalamatl and the Haab meant that any particular day name would not recur for many years. In fact, we note that

$$gcd(260, 365) = 5$$

and so this great cycle of day names will return to its starting point only after

$$\frac{260 \times 365}{5} = 18980 \text{ days}$$

which is just under 52 (modern) years. Since the average life span at that time would only have been about 45 $\tilde{5}$0 years it would have been very unusual for a

Name	No. of previous periods	No. of days
Kin	—	1
Uinal	20 Kin	20
Tun	18 Uinal	360
Katun	20 Tun	7200
Baktun	20 Katun	144×10^3
Pictun	20 Baktun	288×10^4
Calabtun	20 Pictun	576×10^5
Kinchiltun	20 Calabtun	1152×10^6
Alautun	20 Kinchiltun	2304×10^7

Table 4: Maya Time Periods

Mayan to have observed the same day-name appear twice in his or her lifetime. Compare this to the number of times we might see a Tuesday, 7th March.

This system worked well for periods that were short in relation to the lives of Mayans. However for longer periods it had to be elaborated otherwise they would not have been able to tell the difference between one 4 Ahau 8 Cumku and the next one 52 years later, which would have resulted in great confusion over the centuries.

Instead of simply numbering the elapsed years from some initial point the Maya chose to count the number of elapsed days since their very first 4 Ahau 8 Cumku. This date must have held some sort of religious or political significance equivalent to the birth of Christ being used as the reference date for the Gregorian calendar. The number of elapsed days was expressed in an almost vigesimal (base 20) number system. The only exception was in moving from the second position to the third position which required only 18 units of the former (instead of 20) to make up 1 unit of the latter (see Table 4). Denote 4 Ahau 8 Cumku as (4 : 19, 8 : 17) by combining our models of the Tonalamatl and the Haab. Then to find the day-name of a day which is 31741 days later use equations 1 and 2 to evaluate the following

$$\begin{aligned} \text{4 Ahau 8 Cumku} + 31741 \text{ days} &= (4:19, 8:17) + 31741 \\ &= (12:0, 14:16) \\ &= \text{12 Imix, 14 Kayab.} \end{aligned}$$

Next we note that

$$\begin{aligned} 31741 &= (4 \times 7200) + (8 \times 360) + (3 \times 20) + 1 \\ &= 4\text{Katuns} + 8\text{Tuns} + 3\text{Uinals} + 1\text{Kin} \\ &= 4.8.3.1 \end{aligned}$$

using positional notation for brevity. So the full description of this day in Mayan notation would be 4.8.3.1, 12 Imix, 14 Kayab.

The final problem to be considered here is the correspondence between the Mayan calendar and the Gregorian calendar. Because of the destruction of

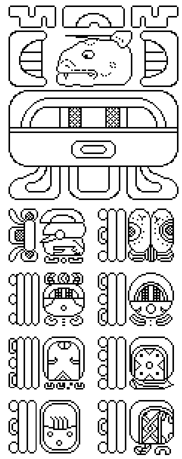

Figure 1: April 22 1985 A.D.

most of the Mayan manuscripts by missionaries this has been a very challenging problem.

The most popular correlation to date, obtained by relating radiocarbon dates of sites to the dates inscribed on monuments, is that made by J. Thompson. He places the Mayan starting date 4 Ahau 8 Cumku on the Gregorian date of 11th August 3114 B.C..

If we assume this to be true then consider the date 22 April 1985 which is 1861894 days after the start of the Mayan calendar. As before equations 1 and 2 show that

$$\begin{aligned} \text{4 Ahau 8 Cumku} + 1861894 \text{ days} &= (4:19, 8:17) + 1861894 \\ &= (12:13, 12:0) \\ &= \text{12 Ix, 12 Pop.} \end{aligned}$$

And since

$$1861894 = (12 \times 144000) + (18 \times 7200) + (11 \times 360) + (16 \times 20) + 14$$

we see that 22 April 1985 A.D. = 12.18.11.16.14, 12 Ix, 12 Pop in the Mayan calendrical system. This would have been inscribed hieroglyphically as shown below.

References

1 Kelley D. H. Deciphering the Maya Script Published 1976 University of Texas Press

2 National Geographic December 1975 Issue