Resolution of the Kandelhardt paradox

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Most paradoxes in mathematics involve disguised division by zero. Here is one which was conveyed to me by my cousin, Andreas Kandelhardt, that does not require such an artifice.

$$\begin{aligned} 25 - 45 &= 16 - 36 \\ 5^2 - 2 \cdot 5 \cdot \frac{9}{2} &= 4^2 - 2 \cdot 4 \cdot \frac{9}{2} \\ 5^2 - 2 \cdot 5 \cdot \frac{9}{2} + \frac{81}{4} &= 4^2 - 2 \cdot 4 \cdot \frac{9}{2} + \frac{81}{4} \\ \left(5 - \frac{9}{2}\right)^2 &= \left(4 - \frac{9}{2}\right)^2 \\ 5 - \frac{9}{2} &= 4 - \frac{9}{2} \\ 5 &= 4 \end{aligned}$$

The step from the third last line to the second last line requires a square root. This has two possible solutions, namely positive or negative. The temptation to equate the two positive roots is overwhelming—and wrong. Equating the opposite sign roots gives the correct answer.

$$\begin{aligned} 25 - 45 &= 16 - 36 \\ 5^2 - 2 \cdot 5 \cdot \frac{9}{2} &= 4^2 - 2 \cdot 4 \cdot \frac{9}{2} \\ 5^2 - 2 \cdot 5 \cdot \frac{9}{2} + \frac{81}{4} &= 4^2 - 2 \cdot 4 \cdot \frac{9}{2} + \frac{81}{4} \\ \left(5 - \frac{9}{2}\right)^2 &= \left(4 - \frac{9}{2}\right)^2 \\ +\left(5 - \frac{9}{2}\right) &= -\left(4 - \frac{9}{2}\right) \\ \frac{1}{2} &= \frac{1}{2} \end{aligned}$$

Ultimately, squaring and taking square roots are not inverses of each other.